Kuta Implicit Differentiation EVENS

(2) $3x^2 + 3y^2 = 2$

$$\frac{d}{dx}[3x^2 + 3y^2] = \frac{d}{dx}[2]$$

$$6x + 6y\frac{dy}{dx} = 0$$

$$6y\frac{dy}{dx} = -6x$$

$$\frac{dy}{dx} = \frac{-6x}{6y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

(4) $4x^2 = 2y^3 + 4y$

$$\frac{d}{dx}[4x^2] = \frac{d}{dx}[2y^3 + 4y]$$

$$8x = 6y\frac{dy}{dx} + 4\frac{dy}{dx}$$

$$8x = (6y + 4)\frac{dy}{dx}$$

$$\boxed{\frac{8x}{6y+4} = \frac{dy}{dx} = \frac{4x}{3y+2}}$$

(6) $1 = 3x + 2x^2y^2$

$\frac{d}{dx}[1] = \frac{d}{dx}[3x + 2x^2y^2]$

requires product rule

$0 = 3 + \frac{d}{dx}[2x^2y^2]$

$0 = 3 + \frac{d}{dx}[2x^2] \cdot y^2 + \frac{d}{dx}[y^2]\, 2x^2$

$0 = 3 + 4xy^2 + 2y\frac{dy}{dx} \cdot 2x^2$

$0 = 3 + 4xy^2 + 4x^2y\frac{dy}{dx}$

$-4x^2y\frac{dy}{dx} = 3 + 4xy^2$

$$\frac{dy}{dx} = \frac{3 + 4xy^2}{-4x^2y} = \frac{-3 - 4xy^2}{4x^2y}$$

(8) $5x^3 + xy^2 = 5x^3y^3$

$\frac{d}{dx}[5x^3+xy^2] = \frac{d}{dx}[5x^3y^3]$

requires product rule

requires product rule

$15x^2 + \frac{d}{dx}[xy^2] = \frac{d}{dx}[5x^3]y^3 + \frac{d}{dx}[y^3] \cdot 5x^3$

$15x^2 + \frac{d}{dx}[x] \cdot y^2 + \frac{d}{dx}[y^2]x = 15x^2y^3 + 3y^2\frac{dy}{dx} \cdot 5x^3$

$15x^2 + 1y^2 + 2y\frac{dy}{dx}x = 15x^2y^3 + 15x^3y^2\frac{dy}{dx}$

$15x^2 + 1y^2 - 15x^2y^3 = 15x^3y^2\frac{dy}{dx} - 2y\frac{dy}{dx}$

$15x^2 + 1y^2 - 15x^2y^3 = \frac{dy}{dx}(15x^3y^2 - 2y)$

$$\boxed{\frac{dy}{dx} = \frac{15x^2 + 1y^2 - 15x^2y^3}{15x^3y^2 - 2y}}$$

(10) $x^2 = (4x^2y^3+1)^2$ Method (1) Chain Rule

$$\frac{d}{dx}(x^2) = \frac{d}{dx}\left[(4x^2y^3+1)^2\right]$$

requires product Rule

$$2x = 2(4x^2y^3+1)\frac{d}{dx}[4x^2y^3+1]$$

$$2x = (8x^2y^3+2)\left[\frac{d}{dx}[4x^2]\cdot y^3 + \frac{d}{dx}[y^3]\cdot 4x^2 + 0\right]$$

$$2x = (8x^2y^3+2)\left[8xy^3 + 3y^2\frac{dy}{dx}\cdot 4x^2\right]$$

* $$2x = (8x^2y^3+2)\left[8xy^3 + 12x^2y^2\frac{dy}{dx}\right]$$

* Method (1) Do NOT expand

$$\frac{2x}{8x^2y^3+2} = 8xy^3 + 12x^2y^2\frac{dy}{dx}$$

$$\frac{2x}{8x^2y^3+2} - 8xy^3 = 12x^2y^2\frac{dy}{dx}$$

$$\frac{1}{12x^2y^2}\left(\frac{2x}{8x^2y^3+2} - 8xy^3\right) = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2x}{96x^4y^5+24x^2y^2} - \frac{8xy^3}{12x^2y^2} = \frac{1}{48x^3y^5+12xy^2} - \frac{2y}{3x}$$

(10) Method (1) cont

from 2

* Method (1) Do expand

$$2x = (8x^2y^3+2)\left(8xy^3+12x^2y^2\frac{dy}{dx}\right)$$

$$2x = 64x^3y^6 + 16xy^3 + 96x^4y^5\frac{dy}{dx} + 24x^2y^2\frac{dy}{dx}$$

$$2x - 64x^3y^6 - 16xy^3 = 96x^4y^5\frac{dy}{dx} + 24x^2y^2\frac{dy}{dx}$$

$$2x - 64x^3y^6 - 16xy^3 = \left(96x^4y^5 + 24x^2y^2\right)\frac{dy}{dx}$$

$$\boxed{\frac{2x - 64x^3y^6 - 16xy^3}{96x^4y^5 + 24x^2y^2} = \frac{dy}{dx}}$$

$$\frac{2x}{2x}\left[\frac{1 - 32x^2y^6 - 8y^3}{48x^3y^5 + 12xy^2}\right] = \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{1 - 32x^2y^6 - 8y^3}{48x^3y^5 + 12xy^2}}$$

(10) Method (2) Expand 1st

$$x^2 = (4x^2y^3+1)^2$$

$$x^2 = (4x^2y^3+1)(4x^2y^3+1)$$

$$x^2 = 16x^4y^6 + 8x^2y^3 + 1$$

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(16x^4y^6 + 8x^2y^3 + 1)$$

requires product rule

$$2x = \frac{d}{dx}[16x^4]y^6 + \frac{d}{dx}[y^6]16x^4 + \frac{d}{dx}[8x^2]\cdot y^3 + \frac{d}{dx}[y^3]\cdot 8x^2 + 0$$

$$2x = 64x^3y^6 + 6y^5\frac{dy}{dx}\cdot 16x^4 + 16xy^3 + 3y^2\frac{dy}{dx}\cdot 8x^2$$

$$2x = 64x^3y^6 + 16xy^3 + 96x^4y^5\frac{dy}{dx} + 24x^2y^2\frac{dy}{dx}$$

$$2x - 64x^3y^6 - 16xy^3 = 96x^4y^5\frac{dy}{dx} + 24x^2y^2\frac{dy}{dx}$$

$$2x - 64x^3y^6 - 16xy^3 = \frac{dy}{dx}(96x^4y^5 + 24x^2y^2)$$

$$\boxed{\frac{dy}{dx} = \frac{2x - 64x^3y^6 - 16xy^3}{96x^4y^5 + 24x^2y^2} = \frac{1 - 32xy^6 - 8y^3}{48x^3y^5 + 12xy^2}}$$

(12) $3x^2+3 = \ln 5xy^2$

$\frac{d}{dx}(3x^2+3) = \frac{d}{dx}(\ln(5xy^2))$

↑ requires chain rule

$6x+0 = \left[\frac{1}{5xy^2}\right] \cdot \frac{d}{dx}[5xy^2]$

↑ requires product rule

$6x = \left[\frac{1}{5xy^2}\right] \cdot \left[\frac{d}{dx}[5x] \cdot y^2 + \frac{d}{dx}[y^2] \cdot 5x\right]$

$6x = \frac{1}{5xy^2} \cdot \left[5y^2 + 2y\frac{dy}{dx} \cdot 5x\right]$

~~Do not expand method~~

* $6x = \frac{1}{5xy^2}\left[5y^2 + 10xy\frac{dy}{dx}\right]$

$(6x)(5xy^2) = 5y^2 + 10xy\frac{dy}{dx}$

$30x^2y^2 = 5y^2 + 10xy\frac{dy}{dx}$

$30x^2y^2 - 5y^2 = 10xy\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{30x^2y^2-5y^2}{10xy} = \frac{6x^2y-y}{2x}$

(12) from *

Do expand method

$$6x = \frac{1}{5xy^2}\left[5y^2 + 10xy\frac{dy}{dx}\right]$$

$$6x = \frac{5y^2}{5xy^2} + \frac{10xy}{5xy^2}\frac{dy}{dx}$$

$$6x = \frac{1}{x} + \frac{2}{y}\frac{dy}{dx}$$

$$6x - \frac{1}{x} = \frac{2}{y}\frac{dy}{dx}$$

$$6xy - \frac{y}{x} = 2\frac{dy}{dx}$$

$$\boxed{\frac{6xy}{2} - \frac{y}{2x} = \frac{dy}{dx}}$$

$$\frac{dy}{dx} = \frac{6xy}{2} - \frac{y}{2x} = \frac{6x^2y}{2x} - \frac{y}{2x} = \frac{6x^2y - y}{2x}$$

(14) $5 = 4x^2 + 5y^2$

$$\frac{d}{dx}[5] = \frac{d}{dx}[4x^2 + 5y^2]$$

$$0 = \frac{d}{dx}[4x^2] + \frac{d}{dx}[5y^2]$$

$$0 = 8x + 10y\frac{dy}{dx}$$

$$-8x = 10y\frac{dy}{dx}$$

$$\frac{-8x}{10y} = \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{-8x}{10y} = \frac{-4x}{5y}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{-4x}{5y}\right]$$ ← requires quotient rule

$$= \frac{\frac{d}{dx}[-4x] \cdot 5y - \frac{d}{dx}[5y] \cdot (-4x)}{(5y)^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-20y + 20x\frac{dy}{dx}}{25y^2}}$$

(14) cont

Recall $\frac{dy}{dx} = \frac{-4x}{5y}$

So $\frac{d^2y}{dx^2} = \frac{-20y + 20x\frac{dy}{dx}}{25y^2}$

$= \frac{-20y + 20x\left(\frac{-4x}{5y}\right)}{25y^2}$

$= \frac{-20y - \frac{80x^2}{5y}}{25y^2}$

$= \frac{-5y}{5y}\left(\frac{\frac{-20y}{1} - \frac{80x^2}{5y}}{25y^2}\right)$

$= \frac{-100y^2 - 80x^2}{125y^3}$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-20y^2 - 16x^2}{25y^3}}$$