

$$\tan \theta =$$

$$\cot \theta =$$

$$\csc \theta =$$

$$\sec \theta =$$

### Pythagorean Identity

Solve the Pythagorean Identity for  $\cos^2 \theta$

Solve the Pythagorean Identity for  $\sin^2 \theta$

Take the Pythagorean Identity and divide every single term by  $\cos^2 \theta$

$$\cos^2 \theta + \sin^2 \theta = 1$$

Solve the above equation for  $\tan^2 \theta$

Take the Pythagorean Identity and divide every single term by  $\sin^2 \theta$

$$\cos^2 \theta + \sin^2 \theta = 1$$

Solve the above equation for  $\cot^2 \theta$

### Some other identities:

$$\sin \theta =$$

$$\cos \theta =$$

$$\tan \theta =$$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

$$-\tan^2 \theta = 1 - \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\cot^2 \theta = \csc^2 \theta - 1$$

$$-\cot^2 \theta = 1 - \csc^2 \theta$$

**Example 1:** Use Trigonometric Identities to write each expression in terms of a single trigonometric identity or a constant.

a.  $\tan \theta \cos \theta$

b.  $\frac{1 - \cos^2 \theta}{\cos^2 \theta}$

c.  $\cos \theta \csc \theta$

d.  $\frac{\sin \theta \sec \theta}{\tan \theta}$

**Example 2:** Simplify the complex fraction.

a.  $\frac{\frac{2}{3}}{\frac{4}{15}}$

b.  $\frac{\frac{4}{5}}{\frac{4}{35}}$

c.  $\frac{\frac{2}{5}}{\frac{3}{5}}$

d.  $\frac{\frac{1}{2}}{2}$

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

$$-\tan^2 \theta = 1 - \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\cot^2 \theta = \csc^2 \theta - 1$$

$$-\cot^2 \theta = 1 - \csc^2 \theta$$

**Example 3:** Simplify the complex fraction.

a.  $\frac{\csc \theta}{\cot \theta}$

b.  $\frac{1 - \cos^2 \theta}{\tan^2 \theta}$

c.  $\frac{\cos \theta \sec \theta}{\tan \theta}$

d.  $\frac{\sin \theta}{\csc \theta}$

Use Trigonometric Identities to write each expression in terms of a single trigonometric identity or a constant.

1.  $\cot \theta \sin \theta$

2.  $\frac{1 - \sin^2 \theta}{\sin^2 \theta}$

1.) \_\_\_\_\_

2.) \_\_\_\_\_

3.  $\sin \theta \sec \theta$

4.  $\frac{\cos \theta \csc \theta}{\cot \theta}$

3.) \_\_\_\_\_

4.) \_\_\_\_\_

Simplify the complex fraction.

5.  $\frac{\sec \theta}{\tan \theta}$

6.  $\frac{1 - \sin^2 \theta}{\cot^2 \theta}$

5.) \_\_\_\_\_

6.) \_\_\_\_\_

7.  $\frac{\sin \theta \csc \theta}{\cot \theta}$

8.  $\frac{\cos \theta}{\sec \theta}$

7.) \_\_\_\_\_

8.) \_\_\_\_\_

**Example 1:** Simplify

a.  $\frac{\tan \theta + \cot \theta}{\tan \theta}$

b.  $\frac{\cos^2 \theta}{1 - \sin \theta}$

c.  $\frac{\sec^2 \theta - 1}{\sec^2 \theta}$

d.  $\tan \theta \csc \theta \cos \theta$

To **VERIFY AN IDENTITY**: Work on each side separately and make sure you don't move things from one side to the other! You can work on both sides at the same time – but you just can't move things from one side to the other.

**Verify the identity.**

**Example 1:**  $\sin \theta \cot \theta \sec \theta = 1$

**Example 2:**  $1 - 2\sin^2 \theta = 2\cos^2 \theta - 1$

**Example 3:** Factor

a.  $a^2 - a^2b$

b.  $x^2 - 2x + 1$

**Example 4:**     *Verify the identity.*

$$\csc^2\theta - \cos^2\theta \csc^2\theta = 1$$

**Example 5:**     **Simplify**

a.  $(\sin\theta - \cos\theta)(\sin\theta + \cos\theta)$

*There are two different ways you can leave this answer! In the notes, leave it in terms of  $\sin^2\theta$ . In the homework, you will be “verifying” and leaving it in terms of  $\cos^2\theta$*

b.  $(\tan\theta + 1)^2$

c.  $\sin^2\theta - 2\sin\theta + 1$

*Simplify the complex fraction.*

1.  $\frac{\csc \theta - \sin \theta}{\csc \theta}$

2.  $\frac{\sin^2 \theta}{1 + \cos \theta}$

1.) \_\_\_\_\_

2.) \_\_\_\_\_

3.  $\frac{\csc^2 \theta - 1}{\csc^2 \theta}$

4.  $\tan \theta \sec \theta \sin \theta$

3.) \_\_\_\_\_

4.) \_\_\_\_\_

*Verify the identity. Both sides should end up being equal, so you will not find these on the answer key.*

5.  $\tan \theta \csc \theta \cos \theta = 1$

6.  $(\sin \theta - \cos \theta)(\sin \theta + \cos \theta) = 1 - 2\cos^2 \theta$

7.  $\frac{\sin \theta}{1 + \cos \theta} \cdot \frac{1 - \cos \theta}{1 - \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$

8.  $\sin^2 \theta (1 + \cot^2 \theta) = 1$

**Verify the identity. Both sides should end up being equal, so you will not find these on the answer key.**

9.  $\frac{\sec \theta - \cos \theta}{\sec \theta} = \sin^2 \theta$

10.  $\frac{\cot \theta \sec \theta}{\csc \theta} = 1$

11.  $\frac{1 + \tan^2 \theta}{\sec \theta} = \sec \theta$

12.  $(1 - \cos \theta)(1 + \cos \theta) = \frac{1}{\csc^2 \theta}$



Example 1: Simplify

a.  $\frac{2}{3} + \frac{1}{4}$

b.  $\frac{1}{\cos \theta} + \frac{1}{\sin \theta}$

c.  $\frac{1}{1 - \cos \theta} + \frac{1}{1 + \cos \theta}$

d.  $\tan \theta - \frac{\sec^2 \theta}{\tan \theta}$

e.  $\frac{\tan \theta}{\cot \theta} + 1$

f.  $\frac{1}{\cos \theta} + \frac{1}{\sin \theta}$

Simplify.

1. 
$$\frac{\sin \theta}{\csc \theta} + \frac{\cos \theta}{\sec \theta}$$

2. 
$$\frac{\csc^2 \theta - 1}{\cot \theta}$$

Verify the identity. Both sides should end up being equal, so you will not find these on the answer key.

3. 
$$\frac{1 + \sec^2 \theta}{\sec^2 \theta} = 1 + \cos^2 \theta$$

4. 
$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{1}{\cos \theta \sin \theta}$$

5. 
$$\sec^2 \theta - \sin^2 \theta \sec^2 \theta = 1$$

6. 
$$\frac{\sin^2 \theta - 2 \sin \theta + 1}{\sin \theta - 1} = \sin \theta - 1$$

7. 
$$\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta}$$

8. 
$$\cot \theta - \frac{\csc^2 \theta}{\cot \theta}$$

## SOLUTIONS

### D1

- |    |               |    |                 |    |               |    |                 |
|----|---------------|----|-----------------|----|---------------|----|-----------------|
| 1. | $\cos \theta$ | 2. | $\cot^2 \theta$ | 3. | $\tan \theta$ | 4. | 1               |
| 5. | $\csc \theta$ | 6. | $\sin^2 \theta$ | 7. | $\tan \theta$ | 8. | $\cos^2 \theta$ |

### D2

- |    |                 |    |                   |    |                 |    |                 |
|----|-----------------|----|-------------------|----|-----------------|----|-----------------|
| 1. | $\cos^2 \theta$ | 2. | $1 - \cos \theta$ | 3. | $\cos^2 \theta$ | 4. | $\tan^2 \theta$ |
|----|-----------------|----|-------------------|----|-----------------|----|-----------------|

### D3

- |    |   |    |               |
|----|---|----|---------------|
| 1. | 1 | 2. | $\cot \theta$ |
|----|---|----|---------------|