

Implicit Differentiation

$$\textcircled{1} \quad 4y^3 = \sqrt[3]{12+36x^4} = (12+36x^4)^{1/3}$$

$$[4y^3] \frac{d}{dx} = [(12+36x^4)^{1/3}] \frac{d}{dx}$$

$$12y^2 \frac{dy}{dx} = \frac{1}{3} (12+36x^4)^{-2/3} [144x^3]$$

$$12y^2 \frac{dy}{dx} = \frac{1}{3} (144x^3) (12+36x^4)^{-2/3}$$

$$12y^2 \frac{dy}{dx} = (48x^3) (12+36x^4)^{-2/3}$$

$$\left(\frac{dy}{dx} = \frac{(48x^3) (12+36x^4)^{-2/3}}{12y^2} \right)$$

$$\left(\frac{dy}{dx} = \frac{4x^3}{y^2} \frac{1}{(12+36x^4)^{2/3}} \right)$$

$$\left(\frac{dy}{dx} = \frac{4x^3}{y^2 \sqrt[3]{(12+36x^4)^2}} \right)$$

$$\textcircled{2} \quad 9x^3 = 2 + 3xy^4$$

$$[9x^3] \frac{d}{dx} = [2 + 3xy^4] \frac{d}{dx}$$

$$27x^2 = [2] \frac{d}{dx} + [3xy^4] \frac{d}{dx}$$

$$27x^2 = 0 + [3x] \frac{d}{dx} y^4 + [3y^4] \frac{d}{dx} x$$

$$27x^2 = 3y^4 + 12y^3 \frac{dy}{dx} \cdot x$$

$$27x^2 = 3y^4 + 12y^3 x \frac{dy}{dx}$$

$$27x^2 - 3y^4 = 12y^3 x \frac{dy}{dx}$$

$$\boxed{\frac{27x^2 - 3y^4}{12y^3 x} = \frac{dy}{dx}}$$

$$\boxed{\frac{dy}{dx} = \frac{9x^2 - y^4}{4xy^3}}$$

$$\textcircled{3} \quad 4y^3 = 2x - 5y^2$$

$$[4y^3] \frac{d}{dx} = [2x - 5y^2] \frac{d}{dx}$$

$$12y^2 \frac{dy}{dx} = [2x] \frac{d}{dx} + [5y^2] \frac{d}{dx}$$

$$12y^2 \frac{dy}{dx} = 2 + 10y \frac{dy}{dx}$$

$$12y^2 \frac{dy}{dx} - 10y \frac{dy}{dx} = 2$$

$$(12y^2 - 10y) \frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2}{12y^2 - 10y}$$

$$\frac{dy}{dx} = \frac{1}{6y^2 - 5y}$$

$$(4) \frac{3y^3}{5x} = 2 + 6x$$

$$\left[\frac{3y^3}{5x} \right] \frac{d}{dx} = [2 + 6x] \frac{d}{dx}$$

$$\frac{\frac{d}{dx}(3y^3) \cdot 5x - \frac{d}{dx}[5x] \cdot 3y^3}{[5x]^2} = \frac{d}{dx}(2) + \frac{d}{dx}[6x]$$

$$\frac{9y^2 \frac{dy}{dx} \cdot 5x - 5 \cdot 3y^3}{25x^2} = 0 + 6$$

$$\frac{45xy^2 \frac{dy}{dx} - 15y^3}{25x^2} = 6$$

$$25x^2 \left(\frac{45xy^2 \frac{dy}{dx} - 15y^3}{25x^2} \right) = (6)(25x^2)$$

$$45xy^2 \frac{dy}{dx} - 15y^3 = 150x^2$$

$$45xy^2 \frac{dy}{dx} = 150x^2 + 15y^3$$

$$\boxed{\frac{dy}{dx} = \frac{150x^2 + 15y^3}{45xy^2} = \frac{10x^2 + y^3}{3xy^2}}$$

$$\textcircled{5} \quad 4x^2 = 3y^2 - 11$$

$$[4x^2] \frac{d}{dx} = [3y^2 - 11] \frac{d}{dx}$$

$$8x = [3y^2] \frac{d}{dx} + [-11] \frac{d}{dx}$$

$$8x = 6y \frac{dy}{dx} + 0$$

$$8x = 6y \frac{dy}{dx}$$

$$\boxed{\frac{8x}{6y} = \frac{dy}{dx}}$$

$$\boxed{\frac{dy}{dx} = \frac{4x}{3y}}$$

tangent slope for (2,3)

$$\boxed{\frac{dy}{dx} = \frac{4(2)}{3(3)} = \frac{8}{9}}$$

tangent line at $x=2$ $y=3$

$$\boxed{y = \frac{8}{9}(x-2) + 3}$$

⑥ Recall $\frac{dy}{dx} = \frac{4x}{3y}$

$$\frac{d^2y}{dx^2} = \left[\frac{4x}{3y} \right] \frac{d}{dx}$$

$$= \frac{\frac{d}{dx}(4x) \cdot 3y - \frac{d}{dx}(3y) \cdot 4x}{[3y]^2}$$

$$= \frac{4 \cdot 3y - 3 \frac{dy}{dx} \cdot 4x}{9y^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{12y - 12x \frac{dy}{dx}}{9y^2}} \quad \text{replace}$$

$$\frac{d^2y}{dx^2} = \frac{12y - 12x \left[\frac{4x}{3y} \right]}{9y^2}$$

$$= \frac{12y - \frac{48x^2}{3y}}{9y^2}$$

$$= \frac{12y - \frac{48x^2}{3y} \cdot \frac{3y}{3y}}{9y^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{36y^2 - 48x^2}{27y^3} = \frac{12y^2 - 16x^2}{9y^3}}$$