Solutions Implicit Differentiation V1

① $5y^3 = \sqrt[3]{15+45x^6}$

$$[5y^3]\frac{d}{dx} = [(15+45x^6)^{1/3}]\frac{d}{dx}$$

$$15y^2\frac{dy}{dx} = \frac{1}{3}(15+45x^6)^{-2/3}(270x^5)$$

$$15y^2\frac{dy}{dx} = \frac{1}{3}(270x^5)(15+45x^6)^{-2/3}$$

$$15y^2\frac{dy}{dx} = (90x^5)(15+45x^6)^{-2/3}$$

$$\boxed{\frac{dy}{dx} = \frac{(90x^5)(15+45x^6)^{-2/3}}{15y^2}}$$

$$\boxed{\frac{dy}{dx} = \frac{6x^5}{y^2}\cdot\frac{1}{\sqrt[3]{(15+45x^6)^2}}}$$

(2) $12x^4 = 9 + 12xy^3$

$[12x^4]\frac{d}{dx} = [9 + 12xy^3]\frac{d}{dx}$

$48x^3 = 9\frac{d}{dx} + [12xy^3]\frac{d}{dx}$

$48x^3 = 0 + \frac{d}{dx}[12x]y^3 + \frac{d}{dx}[12y^3]x$

$48x^3 = 12y^3 + 36y^2 x\frac{dy}{dx}$

$48x^3 - 12y^3 = 36y^2 x\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{48x^3 - 12y^3}{36xy^2}$$

$$\frac{dy}{dx} = \frac{4x^3 - y^3}{3xy^2}$$

(3) $3y^4 = 5x - 6y^3$

$$[3y^4]\frac{d}{dx} = [5x]\frac{d}{dx} + [-6y^3]\frac{d}{dx}$$

$$12y^3\frac{dy}{dx} = 5 + -18y^2\frac{dy}{dx}$$

$$12y^3\frac{dy}{dx} + 18y^2\frac{dy}{dx} = 5$$

$$(12y^3 + 18y^2)\frac{dy}{dx} = 5$$

$$\boxed{\frac{dy}{dx} = \frac{5}{12y^3 + 18y^2}}$$

(4) $\dfrac{8y^4}{9x} = 9 + 4x$

$$\left[\frac{8y^4}{9x}\right]\frac{d}{dx} = [9+4x]\frac{d}{dx}$$

$$\frac{32y^3\frac{dy}{dx}\cdot 9x - 9[8y^4]}{(9x)^2} = [9]\frac{d}{dx} + [4x]\frac{d}{dx}$$

$$\frac{288xy^3\frac{dy}{dx} - 72y^4}{81x^2} = 0 + 4$$

$$288xy^3\frac{dy}{dx} - 72y^4 = 4(81x^2) = 324x^2$$

$$288xy^3\frac{dy}{dx} = 324x^2 - 72y^4$$

$$\frac{dy}{dx} = \frac{324x^2 - 72y^4}{288xy^3} = \frac{9x^2 - 2y^4}{8xy^3}$$

(5) $4x^2 = 3y^2 - 11$

$[4x^2]\frac{d}{dx} = [3y^2 - 11]\frac{d}{dx}$

$8x = [3y^2]\frac{d}{dx} + [-11]\frac{d}{dx}$

$8x = 6y\frac{dy}{dx} + 0$

$8x = 6y\frac{dy}{dx}$

$\frac{8x}{6y} = \frac{6y\frac{dy}{dx}}{6y}$

$\frac{dy}{dx} = \frac{8x}{6y}$

$$\boxed{\frac{dy}{dx} = \frac{4x}{3y}}$$

target slope at $(2, -3)$

$\frac{dy}{dx} = \frac{4(2)}{3(-3)} = \frac{-8}{9}$

$$\boxed{y = -\frac{8}{9}(x-2) - 3}$$

target line at $x = 2$, $y = -3$

(6) $\left[\frac{dy}{dx}\right]\frac{d}{dx} = \left[\frac{4x}{3y}\right]\frac{d}{dx}$

$$\frac{d^2y}{dx^2} = \frac{4 \cdot 3y - 3\frac{dy}{dx} \cdot 4x}{(3y)^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{12y - 12x\frac{dy}{dx}}{3y^2}}$$

$$\frac{d^2y}{dx^2} = \frac{12y - 12x\left[\frac{4x}{3y}\right]}{(3y)^2}$$

$$= \frac{12y - \frac{48x^2}{3y}}{9y^2}$$

$$= \left[\frac{12y - \frac{16x^2}{y}}{9y^2}\right]\frac{y}{y}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{12y^2 - 16x^2}{9y^3}}$$