

FRACTIONAL EXPONENT RULE:

For any real number a and integers n and m : _____

Examples:

a. $16^{1/2} = \sqrt[2]{16} = 4$

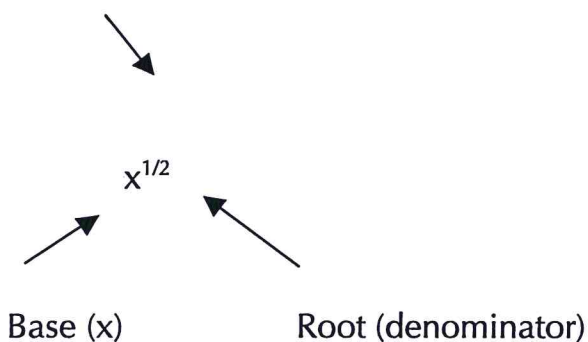
b. $27^{1/3} = \sqrt[3]{27} = 3$

c. $(-8)^{1/3} = \sqrt[3]{-8} = -2$

d. $(16)^{1/4} = \sqrt[4]{16} = 2$

RATIONAL EXPONENTS

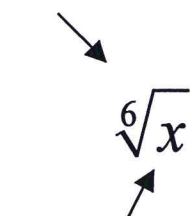
Exponent (numerator)



Exponential Notation	Radical Notation
$x^{1/2}$	
$x^{2/3}$	
$x^{3/4}$	

RADICALS

Index



Radicand

$a^{m/n}$	
$a^{-m/n}$	

RULE:

$$\sqrt{x} = x^{1/2}$$

$$\sqrt[3]{x} = x^{1/3}$$

$$\sqrt[4]{x} = x^{1/4}$$

$$\sqrt[n]{x} = x^{1/n}$$

EXAMPLES: $8^{1/3} = \sqrt[3]{8} = 2$

$$125^{1/3} = \sqrt[3]{125} = 5$$

Evaluate each of the following without the use of a calculator!

1. $100^{1/2} =$	2. $16^{1/4} =$	3. $100,000^{1/5} =$	4. $27^{1/3} =$
5. $81^{1/2} =$	6. $216^{1/3} =$	7. $144^{1/2} =$	8. $1^{1/4} =$
9. $225^{1/2} =$	10. $49^{1/2} =$	11. $1,000^{1/3} =$	12. $25^{1/2} =$

RULE:

$$x^{3/2} = \left(x^{1/2}\right)^3 = \left(\sqrt{x}\right)^3$$

$$x^{m/n} = \left(\sqrt[n]{x}\right)^m$$

EXAMPLES: $8^{2/3} = \left(8^{1/3}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$

$$25^{3/2} = \left(\sqrt{25}\right)^3 = (5)^3 = 125$$

Evaluate each of the following without the use of a calculator!

1. $100^{3/2} =$	2. $16^{3/4} =$	3. $1000^{2/3} =$	4. $25^{3/2} =$
5. $8^{4/3} =$	6. $64^{2/3} =$	7. $64^{3/2} =$	8. $81^{1/2} =$
9. $625^{3/4} =$	10. $49^{3/2} =$	11. $32^{3/5} =$	12. $121^{-1/2} =$

A negative exponent was slipped into that last problem! How did you deal with it?

RULE: $x^{-2} = \frac{1}{x^2}$ $x^{-5} = \frac{1}{x^5}$ $x^{-n} = \frac{1}{x^n}$

EXAMPLES: $8^{-2} = \frac{1}{8^2} = \frac{1}{64}$ $25^{-3/2} = (\sqrt{25})^{-3} = (5)^{-3} = \frac{1}{5^3} = \frac{1}{125}$

Evaluate each of the following without the use of a calculator!

1. $10^{-2} =$

$$\frac{1}{10^2} = \frac{1}{100}$$

2. $16^{-1/2} =$

$$\frac{1}{\sqrt{16}} = \frac{1}{4}$$

3. $1000^{-2/3} =$

$$\frac{1}{\sqrt[3]{1000^2}} = \frac{1}{100}$$

4. $5^{-2} =$

$$\frac{1}{5^2} = \frac{1}{25}$$

5. $125^{-2/3} =$

$$\frac{1}{\sqrt[3]{125^2}} = \frac{1}{25}$$

6. $\left(\frac{1}{4}\right)^{-1/2} =$

$$4^{1/2} = \sqrt{4} = 2$$

7. $49^{-1/2} =$

$$\frac{1}{\sqrt{49}} = \frac{1}{7}$$

8. $81^{-1/2} =$

$$\frac{1}{\sqrt{81}} = \frac{1}{9}$$

9. $6^{-3} =$

$$\frac{1}{6^3} = \frac{1}{216}$$

10. $32^{-3/5} =$

$$\frac{1}{\sqrt[5]{32^3}} = \frac{1}{8}$$

11. $7^{-2} =$

$$\frac{1}{7^2} = \frac{1}{49}$$

12. $\left(\frac{9}{16}\right)^{-1/2} =$

$$\left(\frac{16}{9}\right)^{1/2} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3}$$

Mad Math Minute!

Name: _____

Date: _____

1. $x^{2/3} = \sqrt[3]{x^2}$	2. $(\sqrt[2]{x})^3 = x^{3/2}$	3. $(\sqrt[4]{x})^7 = x^{7/4}$
4. $(\sqrt[3]{x})^4 = x^{4/3}$	5. $x^{1/3} = \sqrt[3]{x}$	6. $100^{1/2} = \sqrt{100} = 10$
7. $y^{1/3} = \sqrt[3]{y}$	8. $(\sqrt[4]{x^3}) = x^{3/4}$	9. $x^{3/2} = \sqrt{x^3}$
10. $49^{1/2} = \sqrt{49} = 7$	11. $x^{1/5} = \sqrt[5]{x}$	12. $(\sqrt{x^4}) = x^{4/2} = x^2$
13. $x^{1/2} = \sqrt{x}$	14. $16^{1/2} = \sqrt{16} = 4$	15. $(\sqrt[5]{x^3}) = x^{3/5}$
16. $x^{5/1} = x^5$	17. $(\sqrt[15]{x})^3 = x^{3/15} = x^{1/5}$	18. $4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$

RADICALS

Warm Up → Simplify the following square root and cube root expressions

1. $\sqrt{-18}$


3. $\sqrt[3]{24}$

2. $\sqrt{48}$

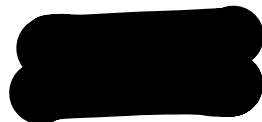
4. $\sqrt[3]{-27}$

$$\text{INDEX} \sqrt{\text{RADICAND}}$$

EXAMPLE ONE → Simplifying square roots with variables

a)  $\sqrt{54x^6y^8z^2}$

b) $\sqrt[3]{-16a^7b^{10}}$



EXAMPLE THREE → Multiplying Radical Expressions

c) $\sqrt{8x^3} \cdot \sqrt{18x}$



EXAMPLE FOUR → Adding and Subtracting Radical Expressions

a) $4\sqrt{18} + 2\sqrt{50}$

b) $\sqrt{48} - 6\sqrt{27} + 4\sqrt{12}$

c) $2\sqrt{75} + 3\sqrt{32} - 8\sqrt{12}$

PRACTICE

$$1. \sqrt{25a^{18}b^{20}}$$

$$= \sqrt{25(a^9b^{10})^2}$$

$$= 5a^9b^{10}$$

$$2. \sqrt{8x^6y^8}$$

$$= \sqrt{4 \cdot 2(x^3y^4)^2}$$

$$= 2\sqrt{2}x^3y^4$$

$$3. \sqrt{x^{11}}$$

$$= x^{11/2}$$

$$4. \sqrt[5]{\frac{x^5}{y^{10}}} = \left(\frac{x^5}{y^{10}}\right)^{1/5}$$

$$= \frac{x}{y^2}$$

$$5. \sqrt[3]{8x^4y^3}$$

$$= \sqrt[3]{(2^3x^3y^3) \cdot 2x^1y^0}$$

$$= 2x^{1/3}y$$

$$6. \sqrt[4]{81x^5y^2z^8}$$

$$= \sqrt[4]{(3^4x^4y^2z^4) \cdot 3x^1y^0z^4}$$

$$= 3x^{1/4}y^{1/2}z^2$$

Simplify using the exponent rules. (No decimals, keep as fractions)

$1. 5^5 \cdot 5^{-12}$ $= 5^{-7} = \frac{1}{5^7}$	$2. \left(\frac{4}{x}\right)^{-2} = \left(\frac{x}{4}\right)^2$ $= \frac{x^2}{16}$	$3. 5^{\frac{3}{2}} \cdot 5^{\frac{1}{4}}$ $= 5^{\frac{6}{4}} \cdot 5^{\frac{1}{4}}$ $= 5^{\frac{7}{4}} = \sqrt[4]{5^7}$
$4. \left(6^{\frac{2}{3}}\right)^{\frac{1}{2}} = 6^{\frac{2}{6}}$ $= \sqrt[6]{36}$	$5. (a^3b^{-6})(a^2b^0)$ $\frac{a^3a^2}{b^6} = \frac{a^5}{b^6}$	$6. 3^{\frac{1}{4}} \cdot 27^{\frac{1}{4}}$ $= 3^{\frac{1}{4}} \cdot (3^3)^{\frac{1}{4}}$ $= 3^{\frac{1}{4}} \cdot 3^{\frac{3}{4}}$ $= 3^{\frac{4}{4}} = 3$
$7. \frac{11^{\frac{2}{5}}}{11^{\frac{4}{5}}}$ $= 11^{\frac{2-4}{5}} = 11^{-2/5}$ $= \frac{1}{11^{2/5}} = \frac{1}{\sqrt[5]{11^2}}$	$8. \frac{6x^2y^{-2}}{2x^{-3}y}$ $= \frac{3x^5}{y^3}$	$9. \frac{xy^9}{3y^{-2}} \cdot \frac{-7y}{14x^4}$ $= \frac{xy^{12}}{6x^5} \cdot \frac{-7xy^{10}}{42x^4}$

<p>1. $\sqrt[3]{3} \cdot \sqrt[3]{9}$</p> $\sqrt[3]{27} = \boxed{3}$	<p>5. $\sqrt[4]{8} \cdot \sqrt[4]{2}$</p> $\sqrt[4]{16} = \boxed{2}$	<p>6. $\frac{\sqrt[5]{64}}{\sqrt[5]{2}}$</p> $= \sqrt[5]{\frac{64}{2}}$ $= \sqrt[5]{32}$ $= \boxed{2}$
<p>7. $6\sqrt[3]{5} + 2\sqrt[3]{5}$</p> $\boxed{8\sqrt[3]{5}}$	<p>9. $7\sqrt{3} - \sqrt{27}$</p> $7\sqrt{3} - \sqrt{9 \cdot 3}$ $7\sqrt{3} - 3\sqrt{3}$ $\boxed{4\sqrt{3}}$	<p>10. $12\sqrt{32} - 6\sqrt{18}$</p> $12\sqrt{16 \cdot 2} - 6\sqrt{9 \cdot 2}$ $= 12(4)\sqrt{2} - 6(3)\sqrt{2}$ $= 48\sqrt{2} - 18\sqrt{2}$ $= \boxed{30\sqrt{2}}$
<p>12. $\sqrt[3]{24} - \sqrt[3]{3}$</p> $\sqrt[3]{8 \cdot 3} - \sqrt[3]{3}$ $2\sqrt[3]{3} - \sqrt[3]{3}$ $\boxed{1\sqrt[3]{3}}$	<p>15. $6\sqrt[3]{32} - 5\sqrt[3]{4}$</p> $6\sqrt[3]{8 \cdot 4} - 5\sqrt[3]{4}$ $6(2)\sqrt[3]{4} - 5\sqrt[3]{4}$ $12\sqrt[3]{4} - 5\sqrt[3]{4}$ $\boxed{7\sqrt[3]{4}}$	<p>4. $\frac{\sqrt{75}}{\sqrt{3}}$</p> $= \frac{\sqrt{25 \cdot 3}}{\sqrt{3}} = \frac{5\sqrt{3}}{\sqrt{3}}$ $= \boxed{5}$
<p>2. $\sqrt{48}$</p> $= \sqrt{16 \cdot 3} = \boxed{4\sqrt{3}}$	<p>13. $\sqrt{81} \sqrt{75}$</p> $9\sqrt{25 \cdot 3}$ $\boxed{45\sqrt{3}}$	<p>14. $\sqrt[3]{27}$</p> $= \frac{3}{(x^2 y^6)^{1/2}}$ $= \frac{3}{x y^3}$

10-2 Skills Practice

Simplifying Radical Expressions

Simplify each expression.

1. $\sqrt{28} = \sqrt{4 \cdot 7} = 2\sqrt{7}$

2. $\sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$

3. $\sqrt{72} = \sqrt{9 \cdot 8} = 3\sqrt{8}$

4. $\sqrt{99} = \sqrt{9 \cdot 11} = 3\sqrt{11}$

5. $\sqrt{2} \cdot \sqrt{10} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

6. $\sqrt{5} \cdot \sqrt{60} = \sqrt{300} = \sqrt{100 \cdot 3} = 10\sqrt{3}$

7. $3\sqrt{5} \cdot \sqrt{5} = 3\sqrt{25} = 3 \cdot 5 = 15$

8. $\sqrt{6} \cdot 4\sqrt{24} = \sqrt{6} \cdot 4\sqrt{4 \cdot 6} = \sqrt{6} \cdot 4 \cdot 2\sqrt{6} = 8\sqrt{36} = 8 \cdot 6 = 48$

9. $2\sqrt{3} \cdot 3\sqrt{15} = 6\sqrt{45} = 6\sqrt{9 \cdot 5} = 18\sqrt{5}$

10. $\sqrt{16b^4} = 4(b^4)^{1/2} = 4b^2$

11. $\sqrt{81a^2d^4} = 9(ad^2)^{1/2} = 9ad^2$

12. $\sqrt{40x^4y^6} = \sqrt{4 \cdot 10} (x^4y^6)^{1/2} = 2\sqrt{10}x^2y^3$

13. $\sqrt{75m^5p^2} = \sqrt{25 \cdot 3} (m^5p^2)^{1/2} = 5\sqrt{3}m^{5/2}p$

14. $\sqrt{\frac{5}{3}} = (\frac{5}{3})^{1/2}$

15. $\sqrt{\frac{1}{6}} = (\frac{1}{6})^{1/2}$

16. $\sqrt{\frac{6}{7}} \cdot \sqrt{\frac{1}{3}} = \sqrt{\frac{6}{21}} = (\frac{6}{21})^{1/2}$

17. $\sqrt{\frac{9}{12}} = (\frac{9}{10})^{1/2}$

18. $\sqrt{\frac{4h}{5}} = (\frac{4h}{5})^{1/2}$

19. $\sqrt{\frac{12}{b^2}} = \frac{\sqrt{4 \cdot 3}}{(b^2)^{1/2}} = \frac{2\sqrt{3}}{b}$

20. $\sqrt{\frac{45}{4m^4}} = (\frac{45}{4m^4})^{1/2}$

