Exponent and Radical Rules (6.1, 6.2) Day 20

Name: ______________________________ Block: __________

Topic	Definition/Rule	Example(s)
Multiplication	$x^a \cdot x^b = x^{a+b}$	
Power to a Power	$(x^a)^b = x^{ab}$	
Power of a Product	$(ab)^n = a^n b^n$	
Zero Exponents	$x^0 = 1$	
Division	$\frac{x^a}{x^b} = x^{a-b}$	
Power of a Quotient	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	
Simplifying	1. 2. 3.	

Simplifying Exponents

Step	Method	Example
1	Label all unlabeled exponents "1"	$\left(\frac{25\, x^{-6}\; y\; (z^{-11})^{2}}{5\, (x^{-2})^{5}\; y^{8}\; z^{2}}\right)^{-2}$
2	Take the reciprocal of the fraction and make the outside exponent positive.	____________
3	Get rid of any inside parentheses.	____________
4	Reduce any fractional coefficients.	____________
5	Move all negatives either up or down. Make the exponents positive.	____________
6	Combine all like bases.	____________
7	Distribute the power to all exponents.	____________

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers. The following properties have the same names as those listed on page 330, but now apply to rational exponents as illustrated.

	Property	Example
1.	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2 + 3/2)} = 5^2 = 25$
2.	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
3.	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
4.	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
5.	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2 - 1/2)} = 4^2 = 16$
6.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

Use properties of exponents to simplify the following expressions.

a. $7^{1/4} \cdot 7^{1/2} =$

b. $(6^{1/2} \cdot 4^{1/3})^2 =$

c. $(4^5 \cdot 3^5)^{-1/5} =$

d. $\frac{5}{5^{1/3}} = \frac{5^1}{5^{1/3}} =$

e. $\left(\dfrac{42^{1/3}}{6^{1/3}}\right)^2 =$

f. $\left(\sqrt[3]{x^2} \bullet \sqrt[6]{x^4}\right)^{-3}$

g. $\dfrac{\sqrt[3]{x} \bullet \sqrt{x^5}}{\sqrt{25x^{16}}}$

h. $12^{1/8} \bullet 12^{5/6} =$

i. $(5^{1/3} \bullet x^{1/4})^3 =$

j. $(2^6 \bullet 4^6)^{-1/6} =$

k. $\dfrac{10}{10^{2/5}} =$

l. $\left(\dfrac{56^{1/4}}{7^{1/4}}\right)^5$

RATIONAL EXPONENTS

QUESTION: What is the square root of a number?

Determine what number fits into the ☐ .

a. $5 \bullet 5 = \square$ → 5 is the ______________ of 25

b. $10 \bullet 10 = \square$ → 10 is the ______________ of 100

c. $x \bullet x = \square$ → _____ is the square root of ______

d. $x^3 \bullet x^3 = \square$ → _____ is the square root of ______

e. $x^9 \bullet x^9 = \square$ → _____ is the square root of ______

QUESTION: What does the **square root of x** mean?

$x^{\square} \bullet x^{\square} = x^1$ → ☐ is the square root of x.

RULE: ____________________________

FRACTIONAL EXPONENT RULE:

For any real number ***a*** and integers ***n*** and ***m***: ______________________________

Examples:

a. $16^{1/2} = \sqrt[2]{16} = 4$

b. $27^{1/3} = \sqrt[3]{27} = 3$

c. $(-8)^{1/3} = \sqrt[3]{-8} = -2$

d. $(16)^{1/4} = \sqrt[4]{16} = 2$

RATIONAL EXPONENTS

Exponent (numerator)

$x^{1/2}$

Base (x)

Root (denominator)

Exponential Notation	Radical Notation
$x^{1/2}$	
$x^{2/3}$	
$x^{3/4}$	

RADICALS

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$\sqrt[6]{x}$

Radicand

$a^{m/n}$	
$a^{-m/n}$	

RULE: $\sqrt{x} = x^{1/2}$ $\sqrt[3]{x} = x^{1/3}$ $\sqrt[4]{x} = x^{1/4}$ $\sqrt[n]{x} = x^{1/n}$

EXAMPLES: $8^{1/3} = \sqrt[3]{8} = 2$ $125^{1/3} = \sqrt[3]{125} = 5$

Evaluate each of the following without the use of a calculator!

1. $100^{1/2} =$	2. $16^{1/4} =$	3. $100,000^{1/5} =$	4. $27^{1/3} =$
5. $81^{1/2} =$	6. $216^{1/3} =$	7. $144^{1/2} =$	8. $1^{1/4} =$
9. $225^{1/2} =$	10. $49^{1/2} =$	11. $1,000^{1/3} =$	12. $25^{1/2} =$

RULE: $x^{3/2} = \left(x^{1/2}\right)^3 = \left(\sqrt{x}\right)^3$ $x^{m/n} = \left(\sqrt[n]{x}\right)^m$

EXAMPLES: $8^{2/3} = \left(8^{1/3}\right)^2 = \left(\sqrt[3]{8}\right)^2 = (2)^2 = 4$ $25^{3/2} = \left(\sqrt{25}\right)^3 = (5)^3 = 125$

Evaluate each of the following without the use of a calculator!

1. $100^{3/2} =$	2. $16^{3/4} =$	3. $1000^{2/3} =$	4. $25^{3/2} =$
5. $8^{4/3} =$	6. $64^{2/3} =$	7. $64^{3/2} =$	8. $81^{1/2} =$
9. $625^{3/4} =$	10. $49^{3/2} =$	11. $32^{3/5} =$	12. $121^{-1/2} =$

A negative exponent was slipped into that last problem! How did you deal with it?

RULE:	$x^{-2} = \frac{1}{x^2}$	$x^{-5} = \frac{1}{x^5}$	$x^{-n} = \frac{1}{x^n}$
EXAMPLES:	$8^{-2} = \frac{1}{8^2} = \frac{1}{64}$	$25^{-3/2} = \left(\sqrt{25}\right)^{-3} = (5)^{-3} = \frac{1}{5^3} = \frac{1}{125}$	

Evaluate each of the following without the use of a calculator!

1. $10^{-2} =$	2. $16^{-1/2} =$	3. $1000^{-2/3} =$	4. $5^{-2} =$
5. $125^{-2/3} =$	6. $\left(\frac{1}{4}\right)^{-1/2} =$	7. $49^{-1/2} =$	8. $81^{-1/2} =$
9. $6^{-3} =$	10. $32^{-3/5} =$	11. $7^{-2} =$	12. $\left(\frac{9}{16}\right)^{-1/2} =$

Mad Math Minute!

Name: ________________________________ Date: ____________

1. $x^{2/3} =$ _______	2. $\left(\sqrt[2]{x}\right)^3 =$ _______	3. $\left(\sqrt[4]{x}\right)^7 =$ _______
4. $\left(\sqrt[3]{x}\right)^4 =$ _______	5. $x^{1/3} =$ _______	6. $100^{1/2} =$ _______
7. $y^{1/3} =$ _______	8. $\left(\sqrt[4]{x^3}\right) =$ _______	9. $x^{3/2} =$ _______
10. $49^{1/2} =$ _______	11. $x^{1/5} =$ _______	12. $\left(\sqrt{x^4}\right) =$ _______
13. $x^{1/2} =$ _______	14. $16^{1/2} =$ _____	15. $\left(\sqrt[5]{x^3}\right) =$ _______
16. $x^{5/1} =$ _______	17. $\left(\sqrt[15]{x}\right)^3 =$ _______	18. $4^{3/2} =$ _______

RADICALS

Warm Up→ Simplify the following square root and cube root expressions

1. $\sqrt{-18}$
2. $\sqrt{48}$
3. $\sqrt[3]{24}$
4. $\sqrt[3]{-27}$

$$\sqrt[INDEX]{RADICAND}$$

EXAMPLE ONE → Simplifying square roots with variables

a) $\sqrt{54x^5y^8z}$

b) $\sqrt[3]{-16a^7b^{10}}$

EXAMPLE TWO → Rationalizing the denominator

a) $\frac{5}{\sqrt[3]{6}}$

b) $\frac{7}{1+2\sqrt{5}}$

EXAMPLE THREE → Multiplying Radical Expressions

c) $\sqrt{8x^3} \bullet \sqrt{18x}$

d) $(1 - \sqrt{3x})(4 + \sqrt{x})$

EXAMPLE FOUR → Adding and Subtracting Radical Expressions

a) $4\sqrt{18} + 2\sqrt{50}$

b) $\sqrt{48} - 6\sqrt{27} + 4\sqrt{12}$

c) $2\sqrt{75} + 3\sqrt{32} - 8\sqrt{12}$

PRACTICE

1. $\sqrt{25a^{18}b^{20}}$

2. $\sqrt{8x^6y^8}$

3. $\sqrt{x^{11}}$

4. $\sqrt[5]{\frac{x^5}{y^{10}}}$

5. $\sqrt[3]{8x^4y^3}$

6. $\sqrt[4]{81x^5y^2z^8}$

Simplify using the exponent rules. (No decimals, keep as fractions)

1. $5^5 \bullet 5^{-12}$	2. $\left(\frac{4}{x}\right)^{-2}$	3. $5^{\frac{3}{2}} \bullet 5^{\frac{1}{4}}$
4. $\left(6^{\frac{2}{3}}\right)^{\frac{1}{2}}$	5. $(a^3b^{-6})(a^2b^0)$	6. $3^{\frac{1}{4}} \bullet 27^{\frac{1}{4}}$
7. $\frac{11^{\frac{2}{5}}}{11^{\frac{4}{5}}}$	8. $\frac{6x^2y^{-2}}{2x^{-3}y}$	9. $\frac{xy^9}{3y^{-2}} \cdot \frac{-7y}{14x^4}$

1. $\sqrt[3]{3} \bullet \sqrt[3]{9}$	5. $\sqrt[4]{8} \bullet \sqrt[4]{2}$	6. $\dfrac{\sqrt[5]{64}}{\sqrt[5]{2}}$
7. $6\sqrt[3]{5} + 2\sqrt[3]{5}$	9. $7\sqrt{3} - \sqrt{27}$	10. $12\sqrt{32} - 6\sqrt{18}$
12. $\sqrt[3]{24} - \sqrt[3]{3}$	15. $6\sqrt[3]{32} - 5\sqrt[3]{4}$	4. $\dfrac{\sqrt{75}}{\sqrt{3}}$
2. $\dfrac{1}{\sqrt[3]{9}}$	13. $\dfrac{5}{\sqrt{3}+7}$	14. $\dfrac{2}{1-4\sqrt{3}}$

EXPONENT PROPERTIES

$$x^{\frac{a}{b}}$$

EXAMPLE FOUR→ Re-write the radical expression in exponential form.

a) $\left(\sqrt[5]{x}\right)^3$ b) $\sqrt[3]{y^4}$ c) $\sqrt{x^5}$ d) $\left(\sqrt[3]{5}\right)^3$

EXAMPLE FIVE→ Simplify the expression without using a calculator.

a) $(-32)^{\frac{2}{5}}$ b) $(-32)^{\frac{7}{5}}$ c) $(8)^{-\frac{4}{3}}$ d) $\left(\frac{8}{27}\right)^{\frac{2}{3}}$