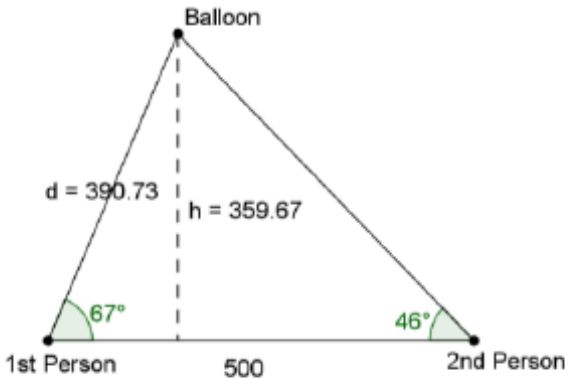
SOLUTIONS Applications of Sine and Cosine Problems

Problem 1

Two people located 500 yards apart have spotted a hot air balloon. The angle of elevation from one person to the balloon is 67°. From the second person to the balloon the angle of elevation is 46°. How high is the balloon when it is spotted?



Let d be the distance between the first person and the balloon. Let h be the height of the balloon in the air.

By the law of sines,

$$\frac{\sin(46°)}{d} = \frac{\sin(67°)}{500}$$
$$d \approx 390.73$$

Then,

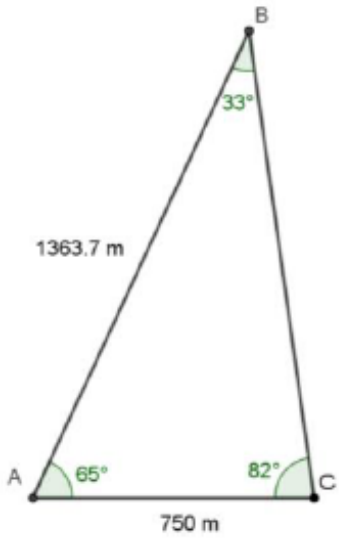
$$\sin(67°) = \frac{h}{390.73}$$
$$h \approx 359.67$$

The balloon is approximately 360 yards in the air when it is spotted.

Inspired by
https://www.engageny.org/file/129481/download/precalculus-m4-topic-b-lesson-10-teacher.pdf?token=CC7xv395

Problem 2

A surveyor needs to determine the distance across a lake between an existing ferry dock at point A and a second dock across the lake at point B. He locates a point C along the shore from the dock at point A that is 750 meters away. He measures the angle at A between the sight lines to points B and C to be $65°$ and the angle at C between the sight lines to points A and B to be $82°$. How far is it from the dock at A and the dock at B?



To find $m\angle B$: $180° - (65° + 82°) = 33°$.

Let d *be the distance from the dock at* A *and the dock at* B.

$$\frac{\sin(33°)}{750} = \frac{\sin(82°)}{d}$$

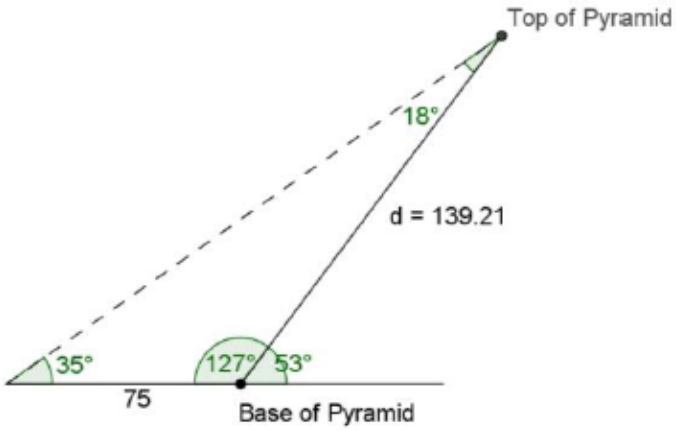
$$d \approx 1363.7$$

The distance between the two docks across the lake is approximately $1,363.7$ *meters.*

Inspired by
https://www.engageny.org/file/129481/download/precalculus-m4-topic-b-lesson-10-teacher.pdf?token=CC7xv395

Problem 3

At the base of a pyramid, a surveyor determines that the angle of elevation to the top is $53°$. At a point 75 meters from the base, the angle of elevation to the top is $35°$. What is the distance from the base of the pyramid up the slanted face to the top?



Let d be the distance from the base to the top of the pyramid.

$$\frac{\sin(18°)}{75} = \frac{\sin(35°)}{d}$$

$$d \approx 139.21$$

The distance is approximately 139 meters.

Inspired by

https://www.engageny.org/file/129481/download/precalculus-m4-topic-b-lesson-10-teacher.pdf?token=CC7xv395

Challenge problem

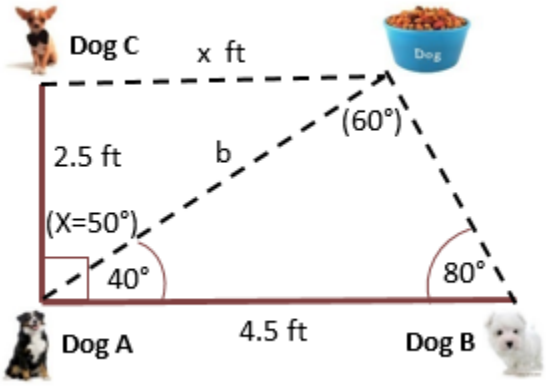
Three dogs are sitting in a kitchen and waiting to get their dog food. It turns out that Dog A is **4.5** feet from Dog B, and Dog C is **2.5**feet from Dog A, as shown in the diagram below (note that the three dogs' positions form a right (**90**°) angle).

The angles formed at Dogs A and B to the dog food are **40**° and **80**°, respectively, as shown in the diagram.

How far is Dog C from the dog food?

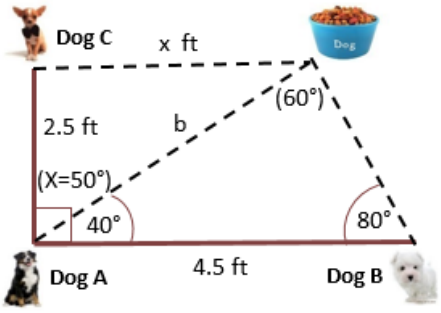
Related figure

Show work that will justify your answers



This one's a little tricky, since we have more than one triangle. We'll first work with the triangle where we have enough information, and then use a common side to solve the parts of the triangle we want.

We can start out using the triangle between Dogs A and B and the dog food, where we have an **ASA** case (**Law of Sines**):

Picture	Math
	Let's use **Law of Sines** to get the distance between Dog A and the food (side **b**). We have an **ASA** case, but we need to find the missing angle (60°) so we can get **b**: $$\frac{\sin(60)}{4.5}=\frac{\sin(80)}{b};\quad b=5.1172$$ (I used more decimal places for better accuracy). Now we have an **SAS** case for the left-most triangle, since we know that angle X = 50° (the sum of 2 angles in a corner = 90°). We can use the **Law of Cosines** to get side **x**: $$x^2=5.1172^2+2.5^2-2(5.1172)(2.5)\cos(50)$$ $$x=\sqrt{5.1172^2+2.5^2-2(5.1172)(2.5)\cos(50)}$$ $$=3.9987\approx 4$$ So the distance between Dog C and the dog food is about **4 feet**.

Inspired by http://www.shelovesmath.com/trigonometry/law-sines-cosines/

State the number of possible triangles that can be formed using the given measurements.

$m\angle C = 63°, b = 9, c = 12$

One triangle

$m\angle B = 33°, a = 27, b = 22$

Two triangles

$m\angle B = 29°, a = 14, b = 19$

One triangle

$m\angle B = 95°, b = 24, a = 5$

One triangle

$m\angle A = 29°, c = 18, a = 17$

Two triangles

$m\angle B = 35°, a = 24, b = 6$

None

Your mathematics teacher states that you only have to do TWO of the problems below.

Which TWO should you pick?

Explain WHY you made your decision: HINT: Pictures could be helpful

Obtuse triangle 1	Obtuse triangle 2	Obtuse triangle 3
$m\angle A = 100$ $a = 12$ $b = 15$ This triangle is impossible because the side opposite of the largest angle is smaller than one of the other sides	$m\angle A = 100$ $a = 15$ $b = 9$ SSA (ONE solution) This triangle already has an OBTUSE angle so it CANNOT have a second triangle based on the given information	$m\angle A = 100$ $b = 15$ $c = 9$ This is a SAS triangle and it must have exactly ONE solution

So as long as you pick obtuse triangle 1 and one of the others with support the level of difficulty is the same. If you pick Obtuse triangle 2 and 3 and omit Obtuse triangle 1 you will do more work than if you selected Triangle 1 and Triangle 2 or Triangle 1 and Triangle 3